

Mean

To find the mean or average of a set of numbers, add them up and divide by the number of numbers.

$$\text{Average} = \frac{\text{Sum of the terms}}{\text{Number of terms}}$$

At times you need to use the mean to find the sum. Then use

$$\text{Sum} = (\text{Average}) \times (\text{Number of terms})$$

To **find a missing number** when you are given the average, **use the sum**.

A few solved examples and some questions to practice

EXAMPLE 1 The arithmetic mean of numbers 7, 9, 0, x and 6 is 5.4. Find the value of x .

SOLUTION Sum of the given numbers = $(7 + 9 + 0 + x + 6) = (22 + x)$

And, there are 5 numbers.

$$\text{arithmetic mean} = \frac{22 + x}{5}$$

$$\frac{22 + x}{5} = 5.4$$

$$\Rightarrow 22 + x = 5.4 \times 5$$

$$\Rightarrow 22 + x = 5.4 \times 5$$

$$\Rightarrow 22 + x = 27$$

$$\Rightarrow x = 5.$$

Hence, the value of x is 5.

EXAMPLE 2 The mean monthly salary of 10 workers in a factory is Rs 4450. One more worker joins the factory. If the monthly salary of the new worker is Rs. 5000, find the mean salary of all the 11 workers in the factory.

SOLUTION Mean salary of 10 workers = Rs 4450.

$$\text{Total salary of 10 workers} = \text{Rs } (4450 \times 10) = \text{Rs } 44500$$

$$\text{Salary of the new worker} = \text{Rs } 5000.$$

$$\text{Total salary of 11 workers} = \left(\frac{44500}{11} \right) = \text{Rs } 4045.45 \text{ per worker.}$$

EXAMPLE 3 The mean of 5 numbers is 21. If one of the numbers is excluded then the mean of the remaining numbers is 22.5. Find the excluded number.

SOLUTION Mean of 5 numbers = 21.

$$\text{Their sum} = (21 \times 5) = 105$$

After a number is excluded, the mean of the remaining 4 numbers = 22.5.

$$\text{Sum of these 4 numbers} = (22.5 \times 4) = 90.$$

□ the number excluded

$$= (\text{sum of 4 numbers}) = (22.5 \times 4) = 90.$$

the number excluded

$$= (\text{sum of 5 numbers}) - (\text{sum of 4 remaining numbers})$$

$$= (105 - 90) = 15$$

Hence the excluded number is 15.

EXAMPLE 4 *A cricketer has mean score of 58 runs in 9 innings. Find out how many runs are to be scored by him in the 10th innings to raise the score to 61.*

Solution Mean score of 9 innings = 58 runs
Total score of 9 innings = (58×9) runs = 522 runs
Mean score required for 10 innings = 61
Total score required for 10 innings $(61 \times 10) = 610$.
Number of runs to be scored in the 10th inning
= (total score of 10 innings) – (total score of 9 innings)
 $(610 - 522) = 88$

Hence, the number of runs to be scored in the 10th inning = 88.

EXAMPLE 5 *The average height of 30 students of a class was found to be 150 cm. It was detected later that 165 cm was wrongly copied as 135 cm for computing the mean. Find the correct mean.*

SOULTION Mean height of 30 students as calculated = 150 cm.
The sum of recorded heights = (150×30) cm = 4500 cm.
The correct sum of the heights of 30 students
= $(4500 + 165 - 135)$ cm = 4530 cm.
The correct mean = $\frac{4530 \text{ cm}}{30} = 151$ cm.

EXAMPLE 6 *The mean of 25 observations is 36. If the mean of the first 13 observations is 32 and that of the last 13 observations is 39, find the 13th observation.*

SOULTION Mean of 25 observations = 36
The sum of 25 observations = $(36 \times 25) = 900$.
Let the 13th observation be x .
Mean of first 13 observations = 32
The sum of the first 13 observations = $(32 \times 13) = 416$.
The sum of the first 12 observations = $(416 - x)$.
The mean of the last 13 observations = 39.
The sum of the last 13 observations = $(39 \times 13) = 507 - x$.
 $(416 - x) + x(507 - x) = 900$

$$\Rightarrow 923 - x = 900$$

$$\Rightarrow x = 923 - 900 = 23$$

Hence, the 13th observation is 23.

EXAMPLE 7 *The mean weight of all the students of a class is 46 kg. The mean weight of boys is 50 kg and that of girls is 40 kg. Find the ratio of the number of boys to the number of girls.*

SOLUTION Let there be m boys and n girls in the class.
Then, the total number of students in the class = $(m + n)$.
Mean weight of m boys = 50 kg
Mean weight of m boys = $(50 \times m)$ kg.
Total weight of m boys = $(50 \times m)$ kg.
Mean weight of n girls = 40 kg
Total weight of n girls = $(40 \times n)$ kg.
Mean weight of $(m + n)$ students = 46 kg.
Total weight of $(m + n)$ students = $46(m + n)$ kg.
Total weight of m boys + total weight of n girls
= total weight of $(m + n)$ students.
 $50m + 40n = 46(m + n)$
 $\Rightarrow 50m + 40n = 46m + 46n$
 $\Rightarrow 4m = 6n$.
 $\Rightarrow \frac{m}{n} = \frac{6}{4} = \frac{3}{2}$.
Hence, the ratio of the number of boys to the number of girls is 3:2.

EXAMPLE 8 *In a school, 85 boys and 35 girls appeared in an examination. The mean marks of boys was 40%, whereas the mean marks of girls was 60%. Determine the average marks percentage of the whole group of students.*

SOLUTION The marks of 85 boys = 40.
The marks of 85 boys = $(85 \times 40) = (85 \times 40) = 3400$.
Mean marks of 35 girls = 60.
Total marks of 35 girls = $(60 \times 35) = 2100$.
Total marks of $(85 + 35)$ students = $(3400 + 2100) = 5500$.
Mean marks of whole group = $\left(\frac{5500}{120}\right) = 45.83$
 \square average marks of the whole group is 45.83%.

MEDIAN

MEDIAN In data arranged in ascending or descending order of magnitude, the value of **the observation falling in the middle** is called the median of the data. If the number of observations is odd, the **middle number is the median**. If the number of observations is even, the average of the two numbers falling in the middle of the group is the median of the group.

Example 1 Find the median of the following data:

18, 37, 24, 59, 41, 26, 63, 45, 57, 29

SOLUTION Arranging the given data in ascending order, we have,

18, 24, 26, 29, 37, 41, 45, 57, 59, 63

Here, $n=10$, which is even.

$$\begin{aligned}\text{Median} &= \frac{1}{2} \cdot \left[\left(\frac{n}{2} \right) \text{th item} + \left(\frac{n}{2} + 1 \right) \text{th item} \right] \\ &= \frac{1}{2} (5\text{th item} + 6\text{th item}) \quad [\because n = 10] \\ &= \frac{1}{2} (37 + 41) = \frac{78}{2} = 39.\end{aligned}$$

Example 2 In the above data, if 37 is replaced by 73, find the new median

SOLUTION

Replacing 37 by 73, and rearranging the data in ascending order, we have,

18, 24, 26, 29, 41, 45, 57, 59, 63, 73.

$$\begin{aligned}\therefore \text{New median} &= \frac{1}{2} (5\text{th item} + 6\text{th item}) \\ &= \frac{1}{2} (41 + 45) = \frac{86}{2} = 43.\end{aligned}$$

EXAMPLE 3 The following data have been arranged in ascending order:

12, 14, 17, 20, 22, x, 26, 28, 32, 36.

If the median of the data is 23, find the value of x .

The given observations in ascending order are:

12, 14, 17, 20, 22, x, 26, 28, 32, 36.

Here $n=10$, which is even.

$$\begin{aligned}\text{Median} &= \frac{1}{2} \cdot \left[\left(\frac{n}{2} \right) \text{th item} + \left(\frac{n}{2} + 1 \right) \text{th item} \right] \\ &= \frac{1}{2} (5\text{th item} + 6\text{th item}) \quad [\because n = 10]\end{aligned}$$

$$\frac{1}{2}(22 + x) = \frac{22 + x}{2}.$$

$$\therefore \frac{22 + x}{2} = 23$$

$$\Rightarrow 22 + x = 46$$

$$\Rightarrow x = 46 - 22 = 24.$$

Thus, the required value of x is 24.

EXAMPLE 4 In the above data, if 32 is replaced by 23, find the new median.

SOLUTION

On replacing 32 by 23, the data in ascending order will be:

12, 14, 17, 20, 22, 23, 24, 26, 28, 36.

$$\text{New median} = \frac{1}{2}(\text{5th item} + \text{6th item})$$

$$= \frac{1}{2}(22 + 23) = \frac{45}{2} = 22.5.$$

Mode

The mode of a set of numbers is the value that appears most often. If numbers are 91, 88, 56, 98, 85, 93, 84, 91, 95. Here 91 is the mode because it appears more often than any other number.

If there is a tie for the most common value in the set, the set has **more** than one mode.

MEASURES OF CENTRAL TENDENCY

1. The mean of 40 observations was 160. It was detected on rechecking that the value 165 was wrongly copied as 125 for the computation of mean. Find the correct mean. (161)
4. Calculate the average age of workers in a factory whose ages are as follows:

Age (in years)	Number of workers
18 – 24	12
24 – 30	16
30 – 36	24
36 – 42	16
42 – 48	8
48 – 54	4

(Ans. 33.3)

5. Find the median of the following data:
12, 17, 3, 14, 5, 8, 7, 15

6. The mean of 5 numbers is 18, if one number is excluded, their mean is 16. Find the excluded number. (26)
7. The mean weight of 60 students of a class is 52.75 kg. If the mean weight of 25 students of this class is 51 kg, find the mean weight of the remaining 35 students of the class. (54 kg)
8. The mean of the marks secured by 25 students of section A of class X is 47 and that of 35 students of section B is 51 and that of 30 students of section C is 63. Find the mean of the marks of the students of 3 sections of class X. (50.5)
9. There are 50 students in a class, of which 20 are girls. The average weight of 20 girls is 45 kg and that of 30 boys is 52 kg. Find the mean weight in kg of the entire class. (49.2 kg)
10. A student obtained 60, 75 and 85 marks respectively in 3 monthly examinations in Physics and 95 marks in the final examination. The three monthly examinations are of equal weight-age whereas the final examination is weighted twice as much as the monthly examinations. Find the mean marks for Physics. (82)
11. The number of patients visiting the diabetic clinic and protein urea clinic in the hospital during April 1989 is given below:

Number of Patients	Number of days attending	
	Diabetic clinic	Protein urea clinic
0 – 10	2	4
10 – 20	8	6
20 – 30	7	5
30 – 40	7	8
40 – 50	4	3
50 – 60	2	4

Which of the two diseases has higher incidence? (28, 29)

12. Find the median of the following frequency distribution:

x:	5	7	9	12	14	17	19	21
f:	6	5	3	6	5	3	2	4

13. The following table represents the marks obtained by a batch of 12 students in a certain class test in statistics and physics. Indicate in which subject is level of achievement higher?

(12)

14. Find the arithmetic mean of first ten natural numbers.

(5.5)

Serial No.	1	2	3	4	5	6	7	8	9	10	11	12
Statistics	53	54	32	30	60	46	28	25	48	72	33	65
Physics	55	41	48	49	27	24	23	20	28	60	43	67

15. 2 candidates A and B seeking admission to a professional college have scored the following marks in various subjects with different weights attached to each subject. Calculate the weighted means and say whose is better of the two:

Subject	Marks of A	Marks of B	Weights
English	80	76	1
Maths	88	90	3
Physics	85	86	2
Chemistry	89	90	2

16. Find the median of the following observations:

46 64 87 41 58 77 35 90 55 92 33.

If 92 are replaced by 99 and 41 by 43, find the new median.

17. In a certain examination, the average grade of all students of a class A is 68.4 and that of all students in class B is 71.2. If the average of both classes combined is 70, find the ratio of the number of students in class A and in class B.

19. Show that the algebraic sum of the deviations of a set of values from their arithmetic mean is zero.

20. Find the mean of the following data:

Marks	No. of students	Marks	No. of students
Below 10	5	Below 60	60
Below 20	9	Below 70	70

Below 30	17	Below 80	78
Below 40	29	Below 90	83
Below 50	45	Below 100	85

(48.414)