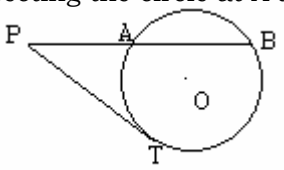
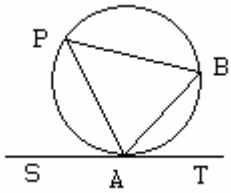
10. TANGENTS TO A CIRCLE

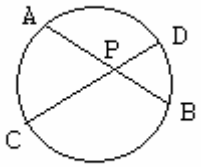
1. If PAB is a secant to a circle intersecting the circle at A and B and PT is a tangent segment, Then PA × PB = PT^2



2. If a chord is drawn through the point of contact of a tangent to a circle, then the angles which this chord makes with the given tangent are equal respectively to the angle formed in the corresponding alternate segment.



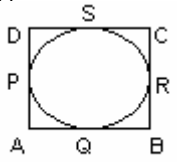
3. If AB and CD are two chords intersecting at a point P inside the circle such that AP = CP, show that AB = CD.



4. In figure, the incircle of △ ABC touches the sides BC, CA and AB at D, E and F respectively. Show that

   AF + BD + CE = AE + BF +CD = ½(perimeter of △ABC)

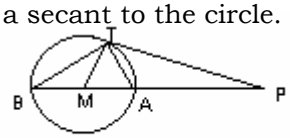
5. If all the sides of a parallelogram touch a circle, show that the parallelogram is a rhombus.



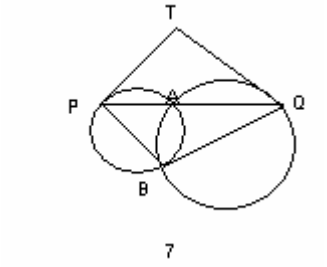
6. In the figure TP is a tangent and PAB is a secant to the circle. If the bisector of ∠ATB intersects AB at M, show that

   (i) ∠PMT = ∠PTM

   (ii) PT = PM



7. Two circles cut at A and B and a straight line PAQ cuts the circles at P and Q. If the tangents at P and Q intersect in T, prove that P,B,Q,T are concyclic.



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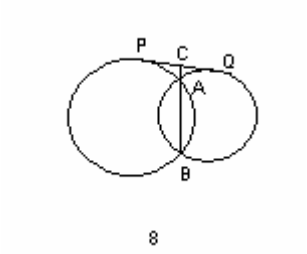
8. If two circles intersect, prove that their common chord when produced bisects their common tangent.



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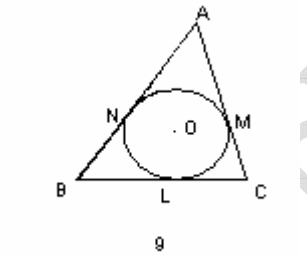
9. If △ABC is isosceles with AB = AC and C(0,r) is the incircle of the △ABC touching BC at L, prove that the point L bisects BC.



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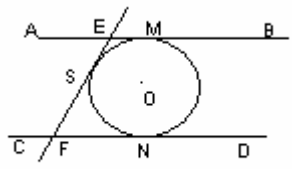
10. Prove that the intercept of a tangent between two parallel tangents to a circle subtend a right angle at the centre.



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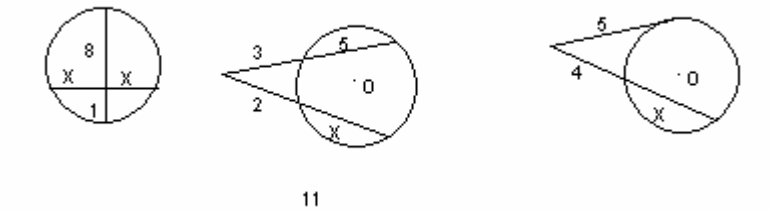
11. Solve for x,



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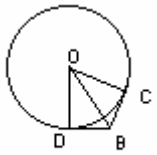
12. Two tangent segments BC, BD are drawn to a circle C(0,r) such that ∠DBC = 120^0. Prove that BO = 2



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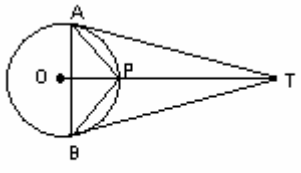
13. TA, TB are tangent segments to a circle from an external point T and OP intersects the angle in P. Prove that AP bisects the angle TAB.



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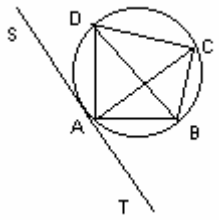
14. With the vertices of a triangle ABC as centres, 3 circles are described each touching the two other externally. If the sides of the triangle are 4 cm, 6 cm and 8 cm, find the radii of the circles.

15. In a cyclic quadrilateral ABCD, the diagonal CA bisects the angle C. Prove that diagonal BD is parallel to the tangent at A to the circle through A, B, C. D.



ANSWERS

1. $\triangle APT$ is similar to TPB.
2. Draw diameter AOM and join MB;
   $\angle ABM = 90^0$; $\angle MAT = 90^0$; $\angle AMG + \angle MAB = \angle MAB + \angle BAT = 90^0$
3. $AP \times PB = CP \times PD$; $PB = PD$
4. $AF = AE; AF + BD + CE = \frac{1}{2}(AB + BC + AC)$
5. $AB = CD$; $AP=AQ$; $(AP+PD) + (RB + CR) = AQ + DS + QB + CS$;
   $AD + BC = AB + CD; 2BC = 2AB$
6. $\angle PTA = \angle PBT$; $\angle BTM=\angle ATM$; EXT. $\angle TMA=\angle MGT+\angle MTB$
7. Join A and B; $\angle TPA = \angle PBA; \angle TQA = \angle QBA$; adding $\angle T$.
8. $CP^2 = CA \times CB; CQ^2 = CA \times CB$
9. $AB = AC; AN = AM; AB - AN = AM$; But BN = BL and CM = CL
10. $\triangle OME$ and $\triangle OSE$ are congruent.

    $\angle OEM = \angle OES$; $\angle OFN = \angle OFS; \angle BFS + \angle DFS = 180^0; \angle OES + \angle OFS = 90^0$.
11. (i) $x^2 = 8 \times 1; x = 2\sqrt{2}$ (ii) $x = 10$ (iii) $x = \frac{9}{4}$
12. In right $\triangle OCB$, $\angle OBC = 60^0$; Cos 60^0 = BC/OB
13. TA = TB; $\triangle TAP$ is congruent to $\triangle TBP$; $\angle TAP = \angle TBP; \angle TBP = \angle PAB$
14. $r_1 + r_2 = 4; r_2 + r_3 = 6; r_3 + r_2 = 8$; 1 cm and 5 cm .

$\angle BAT = \angle BCA; \angle DAS = \angle DCA = \angle DBA$; $\angle TAB = \angle DBA$; alternate angles hence parallel.