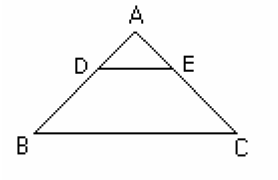
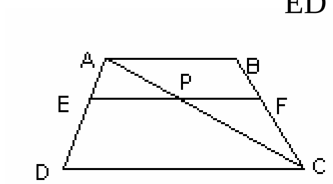
8. SIMILAR TRIANGLES

1. State and prove Basic Proportionality theorem.

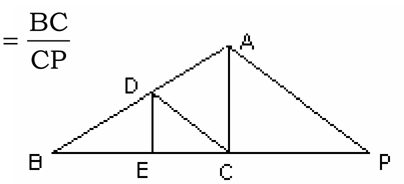


2. In fig. 8.2 EF || AB || DC. Prove that $\frac{AE}{ED} = \frac{BF}{FC}$



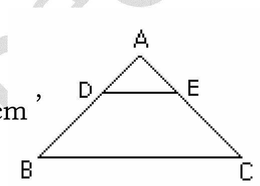
3. In fig. 8.3 DE || AC and DC || AP.

Prove that $\frac{BE}{EC} = \frac{BC}{CP}$

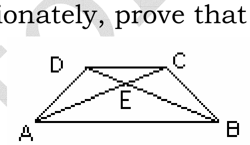


4. In the fig. DE || BC.

$AD = (4x - 3)\,cm$, $AE = (8x-7)\,cm$,
$BD = (3x - 1)\,cm$ and $CE = (5x-3)\,cm$,
find the value of x.

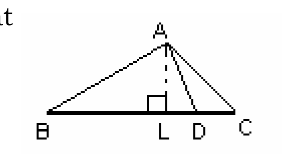


5. If the diagonals of a quadrilateral divide each other proportionately, prove that it is a trapezium.

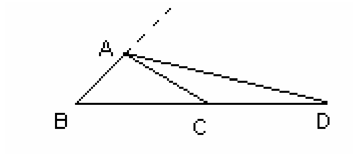


6. In $\triangle ABC$, if AD is the bisector of $\angle A$, prove that

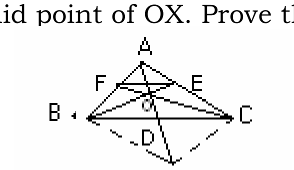
$$\frac{ar\ \triangle ABD}{ar\ \triangle ADC} = \frac{AB}{AC}.$$



7. The bisector of the exterior angle $\angle A$ of a triangle ABC intersects the side BC produced in D. Prove that $\frac{AB}{AC} = \frac{BD}{CD}$.



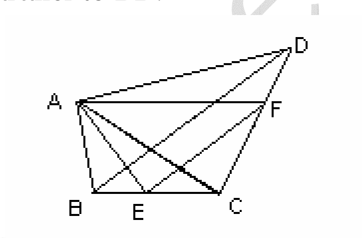
8. In the figure side of $\triangle ABC$ is bisected at D and O is any point on AD. BO and CO produced meet AC and AB in E and F respectively, and AD is produced to X so that D is the mid point of OX. Prove that AO : AX = AF : AB and show that EF || BC.



X

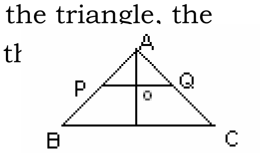
9. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If the median of first triangle is 12.1 cm, find the corresponding median of the other.

10. ABCD is a quadrilateral in which AB = AD. The bisectors of $\angle BAC$ and $\angle CAD$ intersect sides BC and CD at points E and F respectively. Prove that EF is parallel to BD.



11. O is a point inside $\triangle$ ABC. The bisectors of angles AOB, BOC and COA meet the sides AB, BC and CA in points D, E and F respectively. Prove that AD.BE.CF = DB.EC, FA.

12. Prove that if any straight line is drawn parallel to the base of the triangle, the portion intercepted by the sides, is bisected by the median to th

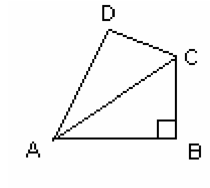


13. In $\triangle$ ABC, AB = AC and D is a point on the side AC such that $BC^2 = AC \times CD$. Prove that BD = BC.

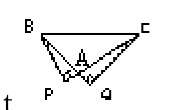
14. ABC is a triangle. PQ is a line segment intersecting AB in P and AC in Q such that $PQ \parallel BC$ and divides $\triangle$ ABC into two parts equal in area. Find $\frac{BP}{AB}$.

15. In quadrilateral ABCD, $\angle B = 90^0$.

$AD^2 = AB^2 + BC^2 + CD^2$, Prove that $\angle ACD = 90^0$.



16. In $\triangle$ ABC, $\angle A$ is obtuse, PB $\perp$ AC and QC $\perp$ AB.

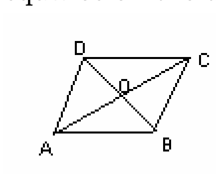


Prove that

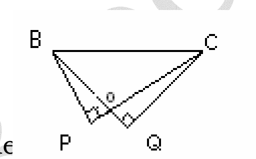
(i) $AB \times AQ = AC \times AP$

(II) $BC^2 = AC \times CP + AB \times BQ$.

17. Prove that sum of the squares on the sides of a rhombus is equal to the sum of the squares on the diagonals.



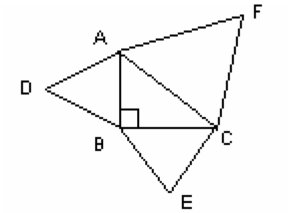
18. In the given figure, a semicircle



diameter and P is a point on AC produced. If BP meets the semicircle on AC produced. If BP meets the semicircle in D, prove that

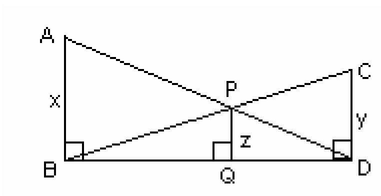
$$AB^2 = (AC \times AP) + (BD \times BP)$$

19. Prove that the equilateral triangles described on the two sides of a right angled triangle are together equal to the equilateral triangle on the hypotenuse in terms of their areas.



20. In the figure, $\angle ABD = \angle CDB = \angle PQB = 90^0$. If AB = x units, CD = y units and PQ = z units, prove that

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$$



ANSWERS

1. $\frac{ar\triangle\ ADE}{ar\triangle\ BDE} = \frac{AD}{BD};\quad \frac{ar\triangle\ ADE}{ar\triangle\ CDE} = \frac{AE}{EC};\quad \frac{AD}{BC} = \frac{AE}{EC}$

2. $\frac{AE}{ED} = \frac{AP}{PC};\quad \frac{BF}{FC} = \frac{AP}{PC}$

3. $\frac{BD}{DA} = \frac{BE}{EC};\quad \frac{BD}{DA} = \frac{BC}{CP}$

4. $x = -\frac{1}{2},\ x = 1$

5. $\frac{DE}{EB} = \frac{DF}{FA};\quad \frac{DF}{FB} = \frac{CE}{EA};\quad FE \parallel DC$

6. $\frac{BD}{DC} = \frac{AB}{AC}$

   $\frac{ar\triangle ABD}{ar\triangle ADC} = \frac{½ \times BD \times AL}{½ \times DC \times AL}$

7. Draw $CE \parallel AD$. $\frac{AO}{AD} = \frac{QO}{CD}$

   Prove AE = AC using alternate and corresponding angles

   $\frac{BD}{CD} = \frac{AB}{AE}$

8. BD = DC and OD = DX

   As diagonals bisect, OBXC is a parallelogram

   $BX \parallel CF$ so $OF \parallel BX$. Sim. $C \times 110E$

   $\frac{AF}{FB} = \frac{AO}{AX};\quad \frac{AO}{OX} = \frac{AE}{AC}$

9. 8.8 cm

10. $\frac{AB}{AC} = \frac{BE}{EC};\quad \frac{AD}{AC} = \frac{FD}{FC};\quad \frac{BE}{EC} = \frac{FD}{FC}$

11. $\frac{AD}{DB} = \frac{OA}{OB};\quad \frac{BE}{EC} = \frac{OB}{OC};\quad \frac{CF}{FA} = \frac{OC}{OA}$

12. $\triangle$ APO and $\triangle$ ABD are similar.

    $\frac{AO}{AD} = \frac{PO}{BD}$

    Sim. $\frac{AO}{AD} = \frac{QO}{CD}$

    But BD = CD

13. $\frac{AC}{BC} = \frac{BC}{DC};\ \angle ACB = \angle BCD;\ \triangle ABC \sim \triangle BDC$

    $\frac{AC}{BC} = \frac{AB}{BD}$

14. $\frac{ar\triangle APQ}{ar\triangle ABC} = \frac{1}{2}$; $\frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

15. $AC^2 = AB^2 + BC^2$; using given condition $AD^2 = AC^2 + CD^2$

16. (i) $BC^2 = AB^2 + AC^2 + 2AB \times AQ$

$BC^2 = AB^2 + AC^2 + 2AC \times AP$

(ii) $BC^2 = AB^2 + AC^2 + (AB \times AQ + AC \times AP)$

$= (AB^2 + AB \times AQ) + (AC^2 + AC \times AP)$

$= AB \times BQ + AC \times CP$

17. $AB^2 = AO^2 + OB^2$

$AB^2 + BC^2 + CD^2 + DA^2 = 2AO^2 + 2CO^2 + 2BO^2 + 2OD^2 = 4AO^2 + 4BO^2$

18. $AB^2 = AC^2 + BC^2 = AC^2 + BP^2 - CP^2 = (AC - CP)(AC + CP) + BP^2$

$= (AC \times CP) - (CP \times AP) + BP^2 = (AC \times AP) + BP^2 - BP \times PD$ $(\because CP \times AP = BP \times PD)$

$= (AC \times AP) + BP(BP - PD)$

19. $\frac{ar\triangle ABD}{ar\triangle ACF} = \frac{AB^2}{AC^2}$; $\frac{ar\ \triangle BCE}{ar\ \triangle ACF} = \frac{BC^2}{AC^2}$;

$ar\ \triangle ABD + ar\ \triangle BCE = ar\ \triangle ACF$

20. $\frac{PQ}{AB} = \frac{QD}{BD}$; $\frac{PQ}{CD} = \frac{BQ}{BD}$; adding $\frac{z}{x} + \frac{z}{y} = 1$