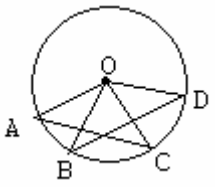
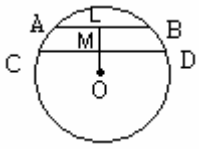
9. CIRCLES

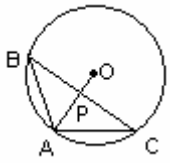
1. In figure $AB \cong CD$. Prove that $\angle A = \angle B$.



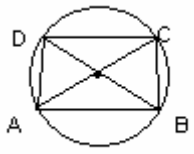
2. Two chords AB and CD of lengths 5 cm and 11 cm are parallel to each other and on the same side of its centre. If the distance between the chords is 3 cm, find the radius of the circle.



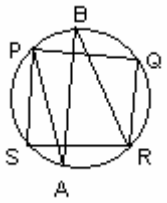
3. In a circle of radius 5 cm, AB = AC = 6 cm. Find the length of the chord BC.



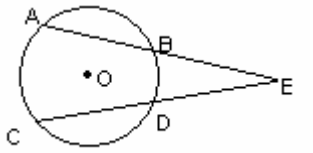
4. In one pair of opposite sides of a cyclic quadrilateral is equal, show that its diagonals are also equal.



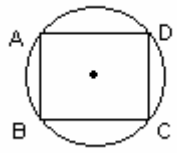
5. Prove that the degree measure of an arc of a circle is twice the angle subtended by it at any point of the alternate segment of the circle with respect to the arc.
6. ABCD is a quadrilateral in which AB = AC = AD.
   Prove that $\angle BAD = 2(\angle CBD + \angle CDB)$.
7. Prove that the bisectors of the opposite angles P and R of a cyclic quadrilateral PQRS intersect the corresponding circle at the points A and B respectively. Prove that AB is a diameter of the circle.



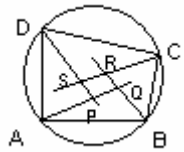
8. AB and CD are equal chords of a circle whose centre is O. When produced these chords meet at E. Prove that EB = ED and EA = EC.



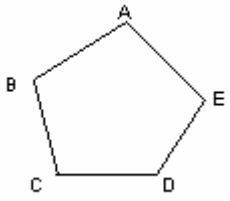
9. If two non parallel sides of a trapezium are equal prove that it is cyclic.



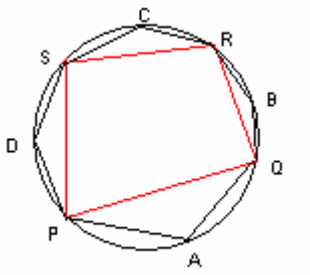
10. Prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also cyclic.



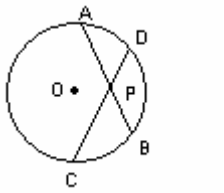
11. Prove that any four vertices of a regular polygon are concyclic.



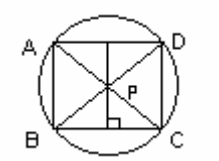
12. The sum of the angle in the four segments exterior to a cyclic quadrilateral is equal to 6 right angles.



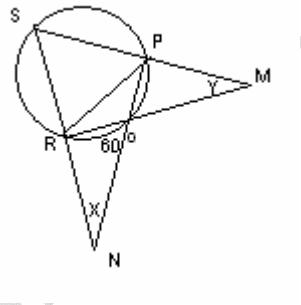
13. In fig. AB and CD are two chords of a circle, intersecting each other at P such that AP = CP. Show that AB = CD.



14. The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection when produced backward bisects the opposite side.



15. In the fig. if y = x, find ∠PNS



ANSWERS

9. CIRCLES

1. $\angle AOB = \angle COD$; $\angle AOC = \angle BOD$; $\triangle AOC \cong \triangle BOD$
2. $OD^2 = OM^2 + MD^2$; $OB^2 = OL^2 + LB^2$; $OM = \frac{5}{2}$ radius = $\frac{\sqrt{146}}{2}$
3. BP = PC; In right $\triangle APB$, $PB^2 = AB^2 - AP^2$

   In $\triangle OPB$, $PB^2 = 5^2 - x^2$ where OP = x. $P^1 = \sqrt{23.04}$
4. CD = AB; ADC = BAD
5.
6. $\angle BAC = 2\angle BDC$
7. $\angle QPS + \angle QRS = 180^0$; $\frac{1}{2}\angle QPS + \frac{1}{2}\angle QRS = 90^0$

   But $\angle BPQ = \angle QRB$; $\angle QPA + \angle QPB = 90^0$
8. $OP \perp AB, OQ \perp CD$; equal chords are equidistant from the centre. Right $\triangle$s OPE and OQE are congruent; PE = QE
9. Draw AM and DN $\perp$ to BC, $\triangle AMB \cong \triangle DNC$,

   $\angle B = \angle C$; $\angle BAM = \angle CDN$; $\angle BAD = \angle CDA$; $2(\angle B + \angle CDA) = 360^0$
10. $\frac{1}{2}\angle A + \frac{1}{2}\angle C = 90^0$; $\frac{1}{2}\angle B + \frac{1}{2}\angle D = 90^0$; $\angle ARB = 180^0 - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right)$;

    $\angle CPD = 180^0 - \left(\frac{1}{2}\angle C + \frac{1}{2}\angle D\right)$; $\angle SRQ + \angle SPQ = 180^0$
11. $\triangle BCD$ congruent to $\triangle ECD$; $\angle CDB = \angle CED$
12. Join PQ, DR; quad. QAPD is cyclic; $\angle QPD + \angle A = 180^0$

    But CD = $\angle SDR + \angle RDQ + \angle QDP$
13. $\triangle PAB$ is similar to $\triangle PCB = \frac{PD}{OB}$; $1 = \frac{PD}{PB}$; adding AP on both sides.
14. $\angle ACD = \angle DBA$; $\angle PLB + \angle PLB = 180^0$;

    $\angle BPL = \angle CPB + \angle CPM = 180^0$; $\angle BPL + \angle CPM = 90^0$;

    $\angle PBL + \angle PBL = \angle BPL + \angle CPM$; $\angle CPM = \angle PBL$;

    $\angle MCP = \angle MPC$; PM = CM; Sim. PM = MD
15. 30^0