Units and Measurement (NCERT text book class 11 Physics)

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Please do not copy the answers. Work out the questions and then compare your answers with the solutions given here.

Note : In stating numerical answers, take care of significant figures.

2.1 Fill in the blanks

(a) The volume of a cube of side 1 cm is equal to ....m$^3$
(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ...(mm)$^2$
(c) A vehicle moving with a speed of 18 km h$^{-1}$ covers.....m in 1 s
(d) The relative density of lead is 11.3. Its density is ....g cm$^{-3}$ or ....kg m$^{-3}$.

Solution:

(a)$10^6$
(b)$1.5 \times 10^4$
(c)$5$
(d)$1.3 \times 10^4$

2.2 Fill in the blanks by suitable conversion of units

(a) 1 kg m$^2$ s$^{-2}$ = ....g cm$^2$ s$^{-2}$
(b) 1 m = ..... ly
(c) 3.0 m s$^{-2}$= .... km h$^{-2}$
(d) $G = 6.67 \times 10^{-11}$ N m$^2$ (kg)$^{-2}$ = .... (cm)$^3$ s$^{-2}$ g$^{-1}$.

Solution:

(a)$10^7$
(b)$1.057 \times 10^{-16}$
(c)$3.9 \times 10^4$
(d)$6.67 \times 10^{-8}$

2.3 A calorie is a unit of heat or energy and it equals about 4.2 J where 1J = 1 kg m$^2$ s$^{-2}$. Suppose we employ a system of units in which the unit of mass equals ( kg, the unit of length equals ( m, the unit of time is ( s. Show that a calorie has a magnitude 4.2 $\cdot (\cdot) \cdot (\cdot) \cdot (\cdot)$ in terms of the new units.

Solution:
In S.I system:
\[ n_1 = 4.2 \]
\[ M_1 = 1\text{ kg} \]
\[ L_1 = 1\text{ m} \]
\[ T_1 = 1\text{ s} \]

In new system:
\[ n_2 = ? \]
\[ M_2 = \alpha \text{ kg} \]
\[ L_2 = \beta \text{ m} \]
\[ T_2 = \gamma \text{ s} \]

Using:
\[ n_{u_1}u_1 = n_{u_2}u_2 \]
\[ \Rightarrow n_2 = \frac{n_{u_1}u_1}{u_2} = n_1 \frac{[M_1^\alpha L_1^\beta T_1^\gamma]}{[M_2^\alpha L_2^\beta T_2^\gamma]} \]

Now
\[ 1 \text{ cal} = 4.2 \text{ J} = 4.2 \text{ kg m}^2 \text{ s}^{-2} \]

So \( a = 1, b = 2, c = -2 \)

Hence
\[ n_2 = 4.2 \left( \frac{1}{\alpha} \right) \left( \frac{1}{\beta} \right) \left( \frac{1}{\gamma} \right)^2 = -4.2 \alpha^{-1} \beta^{-2} \gamma^{-2} \]

or
\[ 1 \text{ cal} = 4.2 \alpha^{-1} \beta^{-2} \gamma^{-2} \text{ in the new system. Hence required statement is proved.} \]

2.4 Explain this statement clearly:
“To call a dimensional quantity ‘large’ or ‘small’ is meaningless without specifying a standard for comparison”. In view of this, reframe the following statements wherever necessary:
(a) atoms are very small objects
(b) a jet plane moves with great speed
(c) the mass of Jupiter is very large
(d) the air inside this room contains a large number of molecules
(e) a proton is much more massive than an electron
(f) the speed of sound is much smaller than the speed of light.
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(a) Atoms are very small objects in comparison to even the tip of a pin.
(b) A jet plane moves faster even than the fastest train on planet.
(c) The mass of Jupiter is very large as compared to earth.
(d) The statement need not be reframed.
(e) The statement need not be reframed.

2.5 A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Solution:
Velocity of light in vacuum, \(c = 1\) new unit of length/s
Time taken by light of sun to reach the earth = 500 s
\(\therefore\) Distance between sun and earth, \(x = c \times t = 1\) new unit/s \(\times\) 500 s = 500 new units of length

2.6 Which of the following is the most precise device for measuring length:
(a) a vernier callipers with 20 divisions on the sliding scale
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale
(c) an optical instrument that can measure length to within a wavelength of light?

(a) Least count of V.C = \(\frac{1}{20}\) MSD - 1 VSD = \(\frac{19}{20}\) MSD = 0.005 cm

(b) Least count of screw gauge = \(\frac{\text{pitch}}{\text{no. of divisions on circular scale}} = 0.001\) cm

(c) Wavelength of light, \(\lambda = 10^{-5}\) cm

Hence, Least count of optical instrument = \(10^{-5}\) cm

Clearly, the optical instrument is the most precise one.

2.7 A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?

Solution:

Magnification of microscope, \(M = 100\)

Observed width, \(w_o = 3.5\) mm

Real width = \(w_{\text{real}}\) (let)

Now: \(M = \frac{w_o}{w_{\text{real}}}\)

\(\Rightarrow w_{\text{real}} = \frac{M}{w_o} \times w_o = \frac{3.5}{100} mm = 3.5 \times 10^{-2} mm\)
2.8 Answer the following:
(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?
(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Solution

(a) Take the thread and wound it in closed turns on the meter scale. Now find the length of thread (coiled) using the meter scale.

\[ d = \frac{\text{Length}}{\text{Number of turns}} = \frac{l}{n} \]

(b) Least count of a screw gauge is given by,
\[ LC = \frac{\text{pitch}}{\text{number of divisions on the circular scale}} \]

Hence by increasing arbitrarily the number of divisions, LC reduces and the accuracy increases (atleast theoretically).

(c) A set of 100 measurements is expected to give a more reliable result than a set of measurements as the random errors get minimised while averaging of the large number of measurements.

2.9 The photograph of a house occupies an area of 1.75 cm² on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m². What is the linear magnification of the projector-screen arrangement.

Solution

Areal Magnification, \( A_m = \frac{\text{Size of the image}}{\text{Size of the object}} = \frac{1.55}{1.75 \times 10^{-4}} = 8857.1 \)

hence, Linear Magnification, \( L_m = \sqrt{8857.1} = 94.1 \)

2.10 State the number of significant figures in the following:
(a) 0.007 m²
(b) 2.64 \times 10^{24} \text{kg}
(c) 0.2370 g cm⁻³
(d) 6.320 J
(e) 6.032 N m⁻²
2.11 The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

**Solution**

For rectangular sheet:

Length, \( l = 4.234 \) m

Breadth, \( b = 1.005 \) m

Thickness, \( h = 0.0201 \) m

Now, Area of sheet, \( A = 2(lb + bh + lh) = 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \)

\[ = 8.7209468 \text{ m}^2 \]

\[ = 8.72 \text{ m} \] (Upto three significant figures)

Volume of sheet, \( V = lbh = 4.234 \times 1.005 \times 0.0201 \text{ m}^3 \)

\[ = 0.0855 \text{ m}^3 \]

2.12 The mass of a box measured by a grocer's balance is 2.300 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box, (b) the difference in the masses of the pieces to correct significant figures?

**Solution:**

Mass of box, \( m = 2.3 \) kg

Mass of one of gold pieces, \( m_1 = 20.15 \) g \( \equiv 0.02015 \) kg

Mass of other gold coin, \( m_2 = 20.17 \) g \( \equiv 0.02017 \) kg

(a) Total mass of box, \( m_{\text{total}} = m + m_1 + m_2 = 2.3 + 0.02015 + 0.02017 = 2.34032 \equiv 2.3 \) kg

(b) Difference in masses of box, \( m_{\text{diff}} = m_2 - m_1 = 0.02017 - 0.02015 = 0.00002 \) kg

2.13 A physical quantity \( P \) is related to four observables \( a, b, c \) and \( d \) as follows:

\[ P = \frac{a b}{(c d)} \]

The percentage errors of measurement in \( a, b, c \) and \( d \) are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity \( P \)? If the value of \( P \) calculated using the above relation turns out to be 3.763, to what value should you round off?
the result?

**Solution:**

\[ P = \frac{a^3 b^2}{\sqrt{c d}} \]

Hence relative error,
\[ \Delta P = 3 \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + \frac{1}{2} \frac{\Delta c}{c} + \frac{\Delta d}{d} = 3(0.01) + 2(0.03) + 0.5(0.04) + (0.02) \]
\[ = 0.13 \equiv 13\% \]

\[ \Rightarrow \Delta P = 0.13 \times 3.763 = 0.49 \]

As error digits comprise two significant digits, \( P = 7.8 \)

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**2.14** A book with many printing errors contains four different formulas for the displacement \( y \) of a particle undergoing a certain periodic motion:

(a) \( y = a \sin 2\pi \frac{t}{T} \)

(b) \( y = a \sin vt \)

(c) \( y = (a/T) \sin t/a \)

(d) \( y = (a/2) (\sin 2\pi t/T + \cos 2\pi t/T) \)

\( (a = \) maximum displacement of the particle, \( v = \) speed of the particle, \( T = \) time-period of motion). Rule out the wrong formulas on dimensional grounds.

**Solution:**

Dimensions on the lhs \([\text{L}]\)

Now dimension on the rhs are:

(a) \([\text{L}]\)

(b) \([\text{L}]\sin[\text{L}]\)

(c) \([\text{LT}^{-1}]\sin[\text{TL}^{-1}]\)

(d) \([\text{L}]\)

Hence by principle of homogeneity of dimensions (b) and (c) are wrong.

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**2.15** A famous relation in physics relates ‘moving mass’ \( m \) to the ‘rest mass’ \( m_0 \) of a particle in terms of its speed \( v \) and the speed of light, \( c \). (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant \( c \). He writes:

\[ \left( \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \right) = 0 \]

\( \square 2 1/2 \).

Guess where to put the missing \( c \).

**Solution:**
According to the principle of homogeneity of dimensions, RHS should have dimensions of mass. So deno is dimensionless. So in the denominator we have to divide the v by c to get the right formula as:

\[ m = \frac{m_0}{(1-(\frac{v}{c})^2)} \]

2.16 The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m³ of a mole of hydrogen atoms? 

**Solution:**

Radius of hydrogen atom, \( d = 0.5 \, \text{Å} = 0.5 \times 10^{-10} \, \text{m} \)

Avagadro's number, \( N_A = 6.023 \times 10^{23} \, \text{atoms} \)

Volume of one atom, \( V = \frac{4}{3} \pi r^3 \)

Hence atomic volume of 1 mole of hydrogen atoms, \( V_T = \frac{4}{3} \pi (0.5 \times 10^{-10})^3 \times 6.023 \times 10^{23} = 3 \times 10^{-7} \, \text{m}^3 \)

2.17 One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large? 

**Solution:**

Volume of one mole of hydrogen at S.T.P., \( V_M = 22.4 \times 10^{-3} \, \text{m}^3 \)

Radius of one molecule of hydrogen atom, \( r = \frac{1}{2} \, \text{Å} = 0.5 \times 10^{-10} \, \text{m} \)

Volume of one molecule of hydrogen, \( V = \frac{4}{3} \pi r^3 \)

Atomic volume of one mole of hydrogen, \( V_m = 6.023 \times 10^{23} \times \frac{4}{3} \pi (0.5 \times 10^{-10})^3 = 3.154 \times 10^{-7} \, \text{m}^3 \)

\[ \text{Atomic Volume} : \text{Molar Volume} = V_m : V_M = 7 \times 10^4 \]

2.18 Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).
Solution:
When a train moves rapidly, the line of sight of a nearby tree changes its direction of motion rapidly. The angle for nearby objects is much greater compared to those at far distance whose angular change is negligible. So, the nearby objects seem to run in opposite direction while far of objects appear to be stationary.

2.19 The principle of ‘parallax’ in section 2.3.1 is used in the determination of distances of very distant stars. The baseline $AB$ is the line joining the Earth’s two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth’s orbit $3 \times 10^{11}$ m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1” (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1” (second) of arc from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

Solution:
Consider AB as base line and O as sun, 
Now parallax $\theta$ of the star is angle made by semi-major axis of earth in a direction ⊥ to direction of Sun.

Now $\theta = \angle AOB = 2\theta$

$\theta = 1$ sec $= 4.85 \times 10^{-6}$ rad

$b = AB = 3 \times 10^{11}$ m

Now, $s = \frac{b}{2\theta} = \frac{3 \times 10^{11}}{2 \times 4.85 \times 10^{-6}} = 3 \times 10^{16}$ m

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2.20 The nearest star to our solar system is 4.29 light years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

Solution:
Distance of the star, $d = 4.29$ light years $= \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}}$ (1 $ly = 9.46 \times 10^{15}$ m; 1 parsec $= 3.08 \times 10^{16}$ m)

$= 1.32$ parsec

Required parallax, $\theta = 2 \times 1.32$ parsec $= 2.64$ parsec

2.21 Precise measurements of physical quantities are a need of science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Solution:
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1. Research related to nano-technology fields involves carrying out measurements of very small orders.
2. Time measurements during nuclear and atomic reactions require precision of very high orders (nanoseconds).
3. Spectroscopical applications in fields of electron microscopy, super computers etc. need very precise measurements of length and time up to fermi levels.

2.22 Just as precise measurements are necessary in science, it is equally important to be able to make rough estimates of quantities using rudimentary ideas and common observations. Think of ways by which you can estimate the following (where an estimate is difficult to obtain, try to get an upper bound on the quantity):
(a) the total mass of rain-bearing clouds over India during the Monsoon
(b) the mass of an elephant
(c) the wind speed during a storm
(d) the number of strands of hair on your head
(e) the number of air molecules in your classroom.

Solutions:
(a) Total mass of rain bearing clouds, \( M = \text{Average rainfall} \times \text{Density of water} \times \text{Area of India} \)
   Thus approximate total mass of rain bearing clouds can be measured.

(b) The mass of an elephant is usually measured with the help of principle of lever. (\( \approx 3000 \text{ kg} \) )

(c) Wind speed can be measured by installation of a wind turbine kind of structure.

(d) Number of hair on head = \( \frac{\text{Area of the head}}{\text{Area of cross section of hair}} \) (\( \approx 10^6 - 10^9 \) )

(e) Number of gas molecules in class room, \( n = \frac{\text{Volume of class room}}{22.4 \times 10^{-3}} \times N_A \) (Avagadro's number)

2.23 The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 10^7 K and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0 \cdot 10^{30} \text{ kg}, radius of the Sun = 7.0 \cdot 10^8 \text{ m}.

Solutions:
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Mass of sun, \( M = 2.0 \times 10^{30} \) kg
Radius of sun, \( R = 7.0 \times 10^8 \) m
Volume of sun, \( V = \frac{4}{3} \pi r^3 \)

Mass density of sun = \( \frac{\text{Mass of Sun}}{\text{Volume of Sun}} = \frac{2.0 \times 10^{30}}{\frac{4}{3} \pi (7.0 \times 10^8)^3} = 1.4 \times 10^3 \) kg m\(^{-3}\)

The density is in the range of solids. The observation can be explained as the sun is made of hot plasma.

2.24 When the planet Jupiter is at a distance of 824.7 million kilometers from the Earth, its angular diameter is measured to be 35.72” of arc. Calculate the diameter of Jupiter.
Solution
Average radius of sodium atom, \( r = \frac{2.5}{2} \) Å = 1.25 × 10\(^{-10}\) m
Volume of sodium atom, \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.25 \times 10^{-10})^3 = 8.18 \times 10^{-30} \) m\(^3\)
Mass of a mole of sodium = 23 grams = 23 × 10\(^{-3}\) kg
Mass of each sodium atom, \( M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} = 3.82 \times 10^{-26} \) kg

\[\therefore\text{Average mass density of sodium atom, } \rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{8.18 \times 10^{-30}} = 4.67 \times 10^3 \) kg m\(^{-3}\)
Density of sodium atom in crystalline phase = 970 kg m\(^{-3}\)
\[\therefore\text{Average density of sodium atom } = \frac{4.67 \times 10^3}{970} = 0.48\]
Hence the densities are of same order of magnitudes as indicated by the value.

Additional Exercises
2.25 A man walking briskly in rain with speed \( v \) must slant his umbrella forward making an angle \( \theta \) with the vertical. A student derives the following relation between \( \theta \) and \( v \): \( \tan \theta = v \) and checks that the relation has a correct limit: as \( v \to 0, \theta \to 0 \), as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.
Solution:
The given relation is \( \tan \theta = v \)
This relation cannot be correct as the dimensions do not match on both sides.
Instead it should be,
\[\tan \theta = \frac{v}{u} \] with the speed of rainfall

2.26 It is claimed that two cesium clocks, if allowed to run for 100 years, free from any
disturbance, may differ by only about 0.02 s. What does this imply for the accuracy of the standard cesium clock in measuring a time-interval of 1 s?

Solution:

Time interval, \( t = 100 \text{ years} \equiv 100 \times 365 \times 86400 \text{ s} \equiv 3.155 \times 10^9 \text{ s} \)

Difference in time, \( \Delta t = 0.2 \text{ s} \)

Fractional error in time, \( \frac{\Delta t}{t} = \frac{0.2}{3.155 \times 10^9} = 6.34 \times 10^{-12} \), which is of order \( 10^{-11} \text{ s} \)

Hence degree of accuracy of standard clock is around \( \frac{1}{10^{11}} \text{s} \).

2.27 Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro’s number and the atomic mass of sodium). Compare it with the density of sodium in its crystalline phase: 970 kg m\(^{-3}\). Are the two densities of the same order of magnitude? If so, why?

Solution;

Average radius of sodium atom, \( r = \frac{2.5}{2} \text{ Å} = 1.25 \times 10^{-10} \text{ m} \)

Volume of sodium atom, \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.25 \times 10^{-10})^3 = 8.18 \times 10^{-30} \text{ m}^3 \)

Mass of a mole of sodium = 23 grams = 23 \( \times 10^{-3} \) kg

Mass of each sodium atom, \( M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} = 3.82 \times 10^{-26} \text{ kg} \)

\[ \therefore \text{Average mass density of sodium atom, } \rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{8.18 \times 10^{-30}} = 4.67 \times 10^3 \text{ kgm}^{-3} \]

Density of sodium atom in crystalline phase = 970 kgm\(^{-3}\)

\[ \therefore \text{Average mass density of sodium atom } \rho = 4.67 \times 10^3 \text{ kgm}^{-3} \]

Density of sodium in crystalline phase = 970 kgm\(^{-3}\)

Hence the densities are of same order of magnitude as indicated by the value.

2.28 The unit of length convenient on the nuclear scale is a fermi: 1 f = 10\(^{-15}\) m. Nuclear sizes obey roughly the following empirical relation:

\[ r = r_0 A^{1/3} \]

where \( r \) is the radius of the nucleus, \( A \) its mass number, and \( r_0 \) is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of sodium nucleus. Compare it with the average mass density of a sodium atom obtained in Exercise 2.27.
Solution:
Radius of nucleus, \( r = r_0 A^{1/3} \)

Hence, Volume of nucleus = \( \frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_0^3 A \)

Mass of nucleus having mass number \( A = A \) amu = \( A \times 1.67 \times 10^{-27} \) kg

\( r_0 = 1.2 \times 10^{-15} \) m

Density of nucleus, \( \rho = \frac{\text{Mass}}{\text{Volume}} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3} \pi (1.2 \times 10^{-15})^3 A} = 2.3 \times 10^{17} \) kgm\(^{-3} \)

As the density is independent of \( A \) so it is same for sodium nuclei also.

Average mass density of sodium obtained in previous solution was \( \rho = 4.67 \times 10^3 \) kgm\(^{-3} \)

Now \( \frac{\rho}{\rho'} = \frac{2.3 \times 10^{17}}{4.67 \times 10^3} = 4.9 \times 10^{13} \)

2.29 A LASER is a source of very intense, monochromatic, and unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after reflection at the Moon’s surface. How much is the radius of the lunar orbit around the Earth?

Solution:
Let the radius of lunar orbit around the earth be \( D \)

Time taken by laser beam from earth to moon and back, \( T = 2.56 \) s

Time taken by laser beam to go from earth to moon, \( t = \frac{T}{2} = 1.28 \) s

Speed of laser light, \( c = 3 \times 10^8 \) m/s

\( S = ct = 3 \times 10^8 \times 1.28 = 3.84 \times 10^6 \) km

2.30 A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects under water. In a submarine equipped with a SONAR the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s\(^{-1} \)).

Solution
Time taken by wave to go from submarine to enemy submarine is, \( t = \frac{77}{2} = 38.5 \) s

Speed of sound, \( v = 1450 \) m/s\(^{-1} \)

Distance of enemy submarine, \( s = vt = 1450 \times 38.5 = 55.825 \) km

2.31 The farthest objects in our Universe discovered by modern astronomers are so distant
that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Solution

**Time** taken, \( t = 3 \times 10^9 \) years = \( 3 \times 10^9 \times 365 \times 86400 \) s

Velocity of light, \( v = 3 \times 10^8 \) ms\(^{-1}\)

\[ \therefore Distance \ of \ quasar \ from \ earth, s = vt = 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 86400 = 2.84 \times 10^{22} \text{ km} \]

2.32 It is a well known fact that during a total solar eclipse the disk of the moon almost completely covers the disk of the Sun. From this fact and from the information you can gather from examples 2.3 and 2.4, determine the approximate diameter of the moon.

Solution:

From examples 2.3 and 2.4, we get that

At solar eclipse, Moon's angular diameter = Sun's angular diameter, \( \alpha = 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad} \)

Earth-moon distance, \( D = 3.84 \times 10^8 \) m

Moon's diameter, \( d = \alpha D = 1920 \times 4.85 \times 10^{-6} \times 3.84 \times 10^8 = 3580 \text{ km} \)

2.33 A great physicist of this century (P.A.M. Dirac) loved playing with numerical values of Fundamental constants of nature. This led him to an interesting observation. Dirac found that from the basic constants of atomic physics (\( c, e, \) mass of electron, mass of proton) and the gravitational constant \( G \), he could arrive at a number with the dimension of time. Further, it was a very large number, its magnitude being close to the present estimate on the age of the universe (~15 billion years). From the table of fundamental constants in this book, try to see if you too can construct this number (or any other interesting number you can think of). If its coincidence with the age of the universe were significant, what would this imply for the constancy of fundamental constants?

**Solution:**

The require number is,

\[ x = \frac{e^4}{16 \pi^2 c^6 m_p m_e^2 c^3 G} \]

e = Charge on an electron

c = Speed of light

G = Gravitational constant

\( m_p \) and \( m_e \) are mass of a proton and electron respectively.

When we put the value of the constants in the equation the value comes out to be approximately equal.
2.4. Solution:
(a) Atoms are very small objects in comparison to even to the tip of a pin.
(b) A jet plane moves faster even than the fastest train on planet.
(c) The mass of Jupiter is very large as compared to Earth.
(d) The statement need not be reframed.
(e) The statement need not be reframed.

2.5. Solution:
Velocity of light in vacuum, \( c = 1 \) new unit of length/s
Time taken by light of sun to reach the earth = 500 s
∴ Distance between sun and earth, \( x = c \times t = 1 \) new unit/s \( \times 500 \) s = 500 new units of length

2.6. Solution:
(a) Least count of V.C = \( \frac{19}{20} \) MSD = \( \frac{1}{20} \) MSD = 0.005 cm
(b) Least count of screw gauge = \( \frac{\text{pitch}}{\text{no. of divisions on circular scale}} \) = 0.001 cm
(c) Wavelength of light, \( \lambda = 10^{-5} \) cm
Hence, Least count of optical instrument = \( 10^{-5} \) cm
Clearly, the optical instrument is the most precise one.

2.7. Solution:
Magnification of microscope, \( M = 100 \)

Observed width, \( w_0 = 3.5 \text{mm} \)

Real width = \( w_{\text{real}} \) (let)

\[
\text{Now: } M = \frac{w_0}{w_{\text{real}}}
\]

\[
\Rightarrow w_{\text{real}} = \frac{M}{w_0} = \frac{3.5}{100} \text{mm} = 3.5 \times 10^{-2} \text{mm}
\]

2.8

Solution

(a) Take the thread and wound it in closed turns on the meter scale. Now find the length of thread (coiled) using the meter scale.

Now diameter of wire is given by,

\[
d = \frac{\text{Length}}{\text{Number of turns}} = \frac{l}{n}
\]

(b) Least count of a screw gauge is given by,

\[
\text{LC} = \frac{\text{pitch}}{\text{number of divisions on the circular scale}}
\]

Hence by increasing arbitrarily the number of divisions, LC reduces and the accuracy increases (atleast theoretically).

(c) A set of 100 measurements is expected to give a more reliable result than a set of measurements as the random errors get minimised while averaging on the large number of measurements.

2.9

Solution

Areal Magnification, \( A_m = \frac{\text{Size of the image}}{\text{Size of the object}} = \frac{1.55}{1.75 \times 10^{-4}} = 8857.1 \)

hence, Linear Magnification, \( L_m = \sqrt{8857.1} = 94.1 \)

2.10

Solution
(a) 1
(b) 3
(c) 4
(d) 4
(e) 4
(f) 4

2.11
Solution
For rectangular sheet:
Length, l = 4.234 m
Breadth, b = 1.005 m
Thickness, h = 0.0201 m

Now, Area of sheet, \( A = 2(lb+bh+lh) = 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234) \)
= 8.7209468 m\(^2\) = 8.72 m\(^2\) (Upto three significant figures)

Volume of sheet, \( V = lbh = 4.234 \times 1.005 \times 0.0201 \text{ m}^3 \)
= 0.0855 m\(^3\)

2.12
Solution:
Mass of box, \( m = 2.3 \text{ kg} \)
Mass of one of gold pieces, \( m_1 = 20.15 \text{ g} = 0.02015 \text{ kg} \)
Mass of other gold coin, \( m_2 = 20.17 \text{ g} = 0.02017 \text{ kg} \)
(a) Total mass of box, \( m_{\text{total}} = m + m_1 + m_2 = 2.3 + 0.02015 + 0.02017 = 2.34032 \approx 2.3 \text{ kg} \)
(b) Difference in masses of box, \( m_{\text{diff}} = m_2 - m_1 = 0.02017 - 0.02015 = 0.00002 \text{ kg} \)

2.13
Solution:
\[ P = \frac{a^2 b^2}{\sqrt{cd}} \]

Hence relative error, \[ \frac{\Delta P}{P} = 3 \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d} = 3(0.01) + 2(0.03) + 0.5(0.04) + (0.02) \]

\[ = 0.13 \times 0.5763 \approx 0.49 \]

As error digits comprise two significant digits, \( P = 7.8 \)

2.14
Solution:
Dimensions on the lhs are [L]
Now dimensions on the rhs are:
(a) [L]
(b) [L][sin[L]]
(c) [LT\(^{-1}\)][sin[TL\(^{-1}\)]]
(d) [L]
Hence by principle of homogeneity of dimensions (b) and (c) are wrong.

2.15
Solution:
According to principle of homogeneity of dimensions, RHS should have dimensions of mass. So denominator should be dimensionless. So in the denominator we have to divide \( v \) by \( c \) to get the right formula as:
\[ m = \frac{m_0}{\left(1 - \left(\frac{v}{c}\right)^2\right)} \]

2.16
Solution:
Radius of hydrogen atom, \( d = 0.5 \, \text{Å} = 0.5 \times 10^{-10} \, \text{m} \)

Avagadro's number, \( N_A = 6.023 \times 10^{23} \) atoms

Volume of one atom, \( V = \frac{4}{3} \pi r^3 \)

Hence atomic volume of 1 mole of hydrogen atoms, \( V_T = \frac{4}{3} \pi (0.5 \times 10^{-10})^3 \times 6.023 \times 10^{23} = 3 \times 10^{-7} \, \text{m}^3 \)

2.17

Solution:
Volume of one mole of hydrogen at S.T.P., \( V_M = 22.4 \times 10^{-3} \, \text{m}^3 \)

Radius of one molecule of hydrogen atom, \( r = \frac{1}{2} \, \text{Å} = 0.5 \times 10^{-10} \, \text{m} \)

Volume of one molecule of hydrogen atom, \( V = \frac{4}{3} \pi r^3 \)

Atomic volume of one mole of hydrogen, \( V_m = 6.023 \times 10^{23} \times \frac{4}{3} \pi (0.5 \times 10^{-10})^3 = 3.154 \times 10^{-7} \, \text{m}^3 \)

Atomic Volume \( \frac{V_M}{V_m} = 7 \times 10^4 \)

2.18

Solution:
When a train moves rapidly, the line of sight of a nearby tree changes its direction of motion rapidly. The angle for nearby objects is much greater compared to those at far distance whose angular change is negligible. So, the nearby objects seem to run in opposite direction while far of objects appear to be stationary.

2.19

Solution:
Consider AB as base line and O as sun,

Now parallax $\theta$ of the star is angle made by semi-major axis of earth in a direction \perpendicular to direction of sun.

Now $\theta = \angle AOB = 2\theta$

$\theta = 1$ sec $= 4.85 \times 10^{-6}$ rad

$b = AB = 3 \times 10^{11}$ m

Now,

$s = \frac{b}{2\theta} = \frac{3 \times 10^{11}}{2 \times 4.85 \times 10^{-6}} = 3 \times 10^{16}$ m
2.20
Solution:
Distance of the star, \( d = 4.29 \) light years = \( \frac{4.29 \times 9.46 \times 10^{15}}{3.08 \times 10^{16}} \) light years (1 light year = 9.46 \( \times \) 10 \( \times \) 15 m; 1 parsec = 3.08 \( \times \) 10 \( \times \) 16 m)
= 1.32 parsec

Required parallax, \( \theta = 2 \times 1.32 \) parsec = 2.64 parsec

2.21
Solution:
1. Research related to nano-technology fields involves carrying out measurements of very small orders.
2. Time measurements during nuclear and atomic reactions require precision of very high orders (nanoseconds).
3. Spectroscopical applications in fields of electron microscopy, super computers etc. need very precise measurements of length and time upto fermi levels.

2.22
Solutions:
(a)
Total mass of rain bearing clouds, \( M = \text{Average rainfall} \times \text{Density of water} \times \text{Area of India} \)
Thus approximate total mass of rain bearing clouds can be measured.

(b) The mass of an elephant is usually measured with the help of principle of lever. \((\approx 3000 \text{ kg})\)

(c) Wind speed can be measured by installation of a wind turbine kind of structure.

(d) Number of hair on head = \( \frac{\text{Area of the head}}{\text{Area of cross section of hair}} \) \((\approx 10^6 - 10^9)\)

(e) Number of gas molecules in class room, \( n = \frac{\text{Volume of class room}}{22.4 \times 10^{-3}} \times N_A \) (Avagadro's number)

2.23
Solutions:
Mass of sun, \( M = 2.0 \times 10^{30} \) kg  
Radius of sun, \( R = 7.0 \times 10^{8} \) m  
Volume of sun, \( V = \frac{4}{3} \pi r^3 \)  

Mass density of sun = \( \frac{\text{Mass of Sun}}{\text{Volume of Sun}} = \frac{2.0 \times 10^{30}}{\frac{4}{3} \pi (7.0 \times 10^{8})^3} = 1.4 \times 10^{3} \) kg m\(^{-3}\)  

The density is in the range of solids. The observation can be explained as the sun is made of hot plasma.

### 2.24 Solution

**Average radius of sodium atom**, \( \tau = \frac{2.5}{2} \) Å = \( 1.25 \times 10^{-10} \) m  

**Volume of sodium atom**, \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.25 \times 10^{-10})^3 = 8.18 \times 10^{-30} \) m\(^3\)  

**Mass of a mole of sodium** = 23 grams = \( 23 \times 10^{-3} \) kg  

**Mass of each sodium atom**, \( M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} = 3.82 \times 10^{-26} \) kg  

\[ \therefore \text{Average mass density of sodium atom}, \rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{8.18 \times 10^{-30}} = 4.67 \times 10^{3} \text{ kgm}^{-3} \]

**Density of sodium atom in crystalline phase** = 970 kgm\(^{-3}\)  

\[ \therefore \frac{\text{Average mass density of sodium atom}}{\text{Density of sodium atom in crystalline phase}} = \frac{4.67 \times 10^{3}}{970} = 0.48 \]

Hence the densities are of same order of magnitude as indicated by the value.

**Distance of Jupiter from earth**, \( d = 824.7 \) million km = \( 8.247 \times 10^6 \) km  

**Angular diameter**, \( \theta = 35.72^\circ \) of arc = \( 35.72 \times 4.85 \times 10^{-6} \) rad  

Let the diameter of Jupiter be \( D \).  

Using the relation, \( \theta = \frac{D}{d} \)  

\[ D = \theta d = 35.72 \times 4.85 \times 10^{-6} \times 8.247 \times 10^6 = 1.429 \text{ km} \]
2.25
Solution:
The given relation is \( \tan \theta = v \)
This relation cannot be correct as the dimensions do not match on both sides.
Instead it should be,
\( \tan \theta = \frac{v}{u} \); \( u \) being the speed of rainfall

2.26
Solution:

Time interval, \( t = 100 \text{ years} = 100 \times 365 \times 86400 \text{ s} = 3.155 \times 10^{9} \text{ s} \)
Difference in time, \( \Delta t = 0.2 \text{ s} \)
Fractional error in time \( \frac{\Delta t}{t} = \frac{0.2}{3.155 \times 10^{9}} = 6.34 \times 10^{-12} \), which is of order \( 10^{-11} \text{ s} \)
Hence degree of accuracy of standard clock is around \( \frac{1}{10^{-11}} \text{ s} \).

2.27
Solution:
Average radius of sodium atom, \( r = \frac{2.5}{2} \text{ Å} = 1.25 \times 10^{-10} \text{ m} \)
Volume of sodium atom, \( V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.25 \times 10^{-10})^3 = 8.18 \times 10^{-30} \text{ m}^3 \)
Mass of a mole of sodium = 23 grams = 23 \( \times 10^{-3} \text{ kg} \)
Mass of each sodium atom, \( M = \frac{23 \times 10^{-3}}{6.023 \times 10^{23}} = 3.82 \times 10^{-26} \text{ kg} \)

\( \therefore \) Average mass density of sodium atom, \( \rho = \frac{M}{V} = \frac{3.82 \times 10^{-26}}{8.18 \times 10^{-30}} = 4.67 \times 10^3 \text{ kgm}^{-3} \)
Density of sodium atom in crystalline phase = 970 kgm\(^{-3}\)

\( \therefore \) Average mass density of sodium atom in crystalline phase = \( \frac{4.67 \times 10^3}{970} = 0.48 \)

Hence the densities are of same order of magnitude as indicated by the value.
2.28
Solution:
Radius of nucleus, $r = r_0 A^{1/3}$
Hence, Volume of nucleus $V = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_0^3 A$

Mass of nucleus having mass number $A = A \ amu = A \times 1.67 \times 10^{-27} \ kg$

$r_0 = 1.2 \ \text{fm} = 1.2 \times 10^{-15} \ m$

Density of nucleus, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3} \pi (1.2 \times 10^{-15})^3 A}$

As the density is independent of $A$ so it is same for sodium nuclei also. Average mass density of sodium obtained in previous solution was $\rho_1 = 4.67 \times 10^3 \ \text{kgm}^{-3}$

Now, $\frac{\rho}{\rho_1} = \frac{2.3 \times 10^7}{4.67 \times 10^3} = 4.9 \times 10^3$

2.29
Solution:
Let the radius of lunar orbit around the earth be $D$

Time taken by laser beam from earth to moon and back, $T = 2.56 \ s$

Time taken by laser beam to go from earth to moon, $t = \frac{T}{2} = 1.28 \ s$

Speed of laser light, $c = 3 \times 10^8 \ \text{ms}^{-1}$

$S = ct = 3 \times 10^8 \times 1.28 = 3.84 \times 10^8 \ km$

2.30
Solution

Time taken by wave to go from submarine to enemy submarine is, $t = \frac{77}{2} = 38.5 \ s$

Speed of sound, $v = 1450 \ \text{ms}^{-1}$

Distance of enemy submarine, $s = vt = 1450 \times 38.5 = 55.825 \ km$

2.31
Solution

$Time$ taken, $t = 3 \times 10^9 \ \text{years} = 3 \times 10^9 \times 365 \times 86400 \ s$

Velocity of light, $v = 3 \times 10^8 \ \text{ms}^{-1}$

$\therefore$ Distance of quasar from earth, $s = vt = 3 \times 10^8 \times 3 \times 10^9 \times 365 \times 86400 = 2.84 \times 10^{22} \ km$
2.32
Solution:
From examples 2.3 and 2.4, we get that
At solar eclipse, Moon’s angular diameter = Sun’s angular diameter, \( \alpha = 1920'' = 1920 \times 4.85 \times 10^{-6} \text{ rad} \)
Earth-moon distance, \( D = 3.84 \times 10^8 \text{ m} \)
Moon’s diameter, \( d = \alpha D = 1920 \times 4.85 \times 10^{-6} \times 3.84 \times 10^8 = 3580 \text{ km} \)

2.33
Solution:
The require number is,
\[
x = \frac{e^A}{16 \pi^2 \epsilon_0^2 m_p m_e^2 c^3 G}
\]
where 
- \( e \) = Charge on an electron
- \( c \) = Speed of light
- \( G \) = Gravitational constant
- \( m_p \) and \( m_e \) are mass of a proton and electron respectively.

When we put the value of the constants in the equation the value comes out to be approximately equal to