Relations and Functions Assignment

QUESTION 1 (i).In the given mapping , is f a not a bijection, give reason for your answer .

$$f : \mathbf{N} \longrightarrow \mathbf{N} \text{ is defined by } f(x) = 2x \ \forall x \in \mathbf{N}$$

QUESTION 1 (ii).:TheNumber of binary operations on the set {1, 2} are_____

QUESTION **2(i).** If f(x) = x + 7 and g(x) = x – 7, x ∈ R, find (fog)(7).

QUESTION 2 (ii).Find the number of all onto functions from the set {1, 2, 3, ... , 10} to itself.

QUESTION 3.Show that the Relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$ is an equivalence relation. Where the relation R is given by
$\{(a,b) : |a-b|$ is a multiple of 4 $\}$.

QUESTION 4: Prove that the Greatest Integer Function $f : \mathbf{R} \to \mathbf{R}$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

QUESTION 5:
Show that the Signum Function $f : \mathbf{R} \to \mathbf{R}$, given by

$$f(x) = \begin{cases} 1, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{ if } x < 0 \end{cases}$$

is neither one-one nor onto.

QUESTION 6:
Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

QUESTION 7:
Let A = **R** – {3} and B = **R** – {1}. Consider the function f: A→B defined by

$$f(x) = \frac{x-2}{x-3}$$

Is f one-one and onto? Justify your answer.

QUESTION 8:

Let f, g and h be functions from **R** to **R**. Show that

$(f + g)oh = foh + goh$

$(f \cdot g)oh = (foh) \cdot (goh)$

QUESTION 9:

State with reason whether following functions have inverse

(i) $f : \{1, 2, 3, 4\} \rightarrow \{10\}$ with

$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g : \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii) $h : \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

QUESTION 10:

Show that $f : [-1, 1] \rightarrow$ **R**, given by $f(x) = \dfrac{x}{x+2}$ is one-one. Find the inverse of the function $f : [-1, 1] \rightarrow$ Range f.

QUESTION 11:

Consider $f : \mathbf{R}^+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{(y - 4)}$, where $\mathbf{R}_+$ is the set of all non-negative real numbers.

QUESTION 12: Let A = Q x Q , Q being the set of rationals . Let '*' be a binary operation on A , defined by (a, b) * (c , d) = (ac , ad + b) . Show that

(i) '*' is not commutative (ii) '*' is associative

(iii The Identity element w.r.t '*' is (1 , 0)

QUESTION 13:Let '*'be a binary operation on the set { 0,1,2,3,4,5} and

$$a*b = \begin{cases} a+b \text{ if } & a+b<6 \\ a+b-6 \text{ if } & a+b \geq 6 \end{cases}$$

QUESTION 14 A relation R on the set of complex numbers is defined by

$$z_1 R z_2 = \frac{z_1 - z_2}{z_1 + z_2}$$

Show that R is an equivalence relation.

QUESTION 15:f: R $\rightarrow R$ Is the function f(x) $= 9x^3$ injective?