10. Vector Algebra

QUESTION 1(i): For what value of λ, are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ orthogonal

QUESTION 1(ii):

If $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$, find the projection of $\vec{b}$ on $\vec{a}$

QUESTION 2(i):

Find the position vector of a point A such that $\vec{OA}$ is inclined to OX at 60^o and to OY at at 45^o and $|OA| = 10$ units.

QUESTION 2(ii): Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

QUESTION 3(i):

Find the value of 'α' for which $\alpha(\hat{i} + \hat{j} + k)$ is a unit vector.

QUESTION 3(ii):

If $\vec{a}$ is a vector of magnitode 5 pointing eastwards and $\vec{b}$ is a vector pointing westward of magnitude 10 .

What is the magnitude and direction of the vector $\vec{a} - \vec{b}$?

QUESTION 4:

Show that the vectors If $\vec{a} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} - 4\hat{k}, \vec{c} = 3\hat{i} + 2\hat{j} - 1\hat{k}$ are linearly independent.

QUESTION 5:

If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 3, |\vec{b}| = 5$ & $|\vec{c}| = 7$, show that the angle between $\vec{a}$ and $\vec{b}$ is 60°.

QUESTION 6:

Using vectors, prove that in a ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Where a, b and c are lengths of the sides opposite to the angle A, B and C of ΔABC respectively.

QUESTION 7:

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Ifa , b, c are the position vectors of the vertices A, B, C of a Δ ABC respectively. Find an expression for the area of Δ ABC and hence deduce the condition for the points A, B, C to be collinear.

QUESTION 8:

Show that the points A, B, C with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right triangle. Also find the remaining angles of the triangle.

QUESTION 9:

In a regular hexagon ABC DEF, if $\vec{AB}=\vec{a}$ and $\vec{BC}=\vec{b}$, then express $\vec{CD}$, $\vec{DE}$, $\vec{EF}$, $\vec{FA}$, $\vec{AC}$, $\vec{AD}$, $\vec{AE}$ and $\vec{CE}$ in terms of $\vec{a}$ and $\vec{b}$.

QUESTION 10:

If are the dia gonals of a parallelogram with sides , $\vec{a}$ and $\vec{b}$ ind the area of parallelogram in terms of and hence find the area with $\vec{d_1}=i+2\hat{j}+3\hat{k}$ and $\vec{d_2}=3i-2\hat{j}+k$.

QUESTION 11:

Find the components and magnitude of the vector $\vec{PQ}$, where P(-1,-2,4) and Q(2,0,-2).

QUESTION 12:

Find λ if the vectors $\vec{a}=\hat{i}-\lambda\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}-5\hat{j}+2\hat{k}$ are perpendicular to each other.

QUESTION 13(i):

For what values of **'λ',** are the vectors (2î - 3 ĵ) and (λî - 6ĵ) parallel

QUESTION 13(ii):

Find the area of a triangle having the points A(1, 1, 1), B(1, 2, 3)and C(2, 3, 1) as its vertices.

QUESTION 14:

If $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$, then show that $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular to each other.

QUESTION 15:

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors , such that $|\vec{a}|=3, |\vec{b}|=4, |\vec{c}|=5.$

Each of the three vectors is perpendicular to the sum of the other two vectors . Find the value of $|\vec{a}+\vec{b}+\vec{c}|$.